

Overview

Successes with Gurobi's Unique Features and Expert Insights

Gurobi Days Korea 2024

Steven Edwards, Ph.D.
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Yes, raw speed is important



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But, don't forget to use the right tools



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Agenda

Team of Experts

Expert Insights

Tool 1

Multi-objective Optimization

Tool 2

No Relaxation Heuristic

Tool 3

Bilinear Constraints

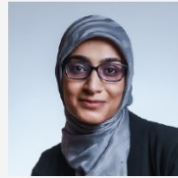
Expert Insights

Unlocking key insights for the water industry

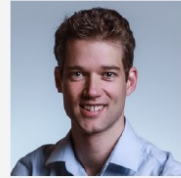
Member of our Team of Experts



Dr. Sonja Mars
Director of
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Support



Dr. Maliheh Aramon
Optimization
Engineer



Dr. Simon Bowly
Optimization
Engineer



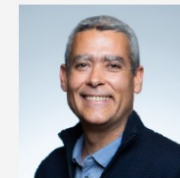
Dr. Gwyneth Butera
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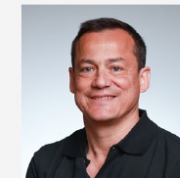
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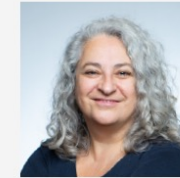
Dan Jeffrey
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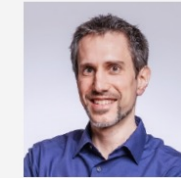
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Dr. Elisabeth Rodriguez Heck
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Dr. Mario Ruthmair
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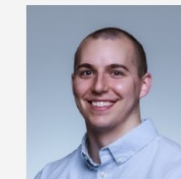
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Optimization Engineer



Dr. Dan Steffy
Optimization Engineer



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Smart experts, many many servers

- We have our own private cluster with HUNDREDS of high-performance servers.
- We have the people and the infrastructure to support you.

Customer: Water Corporation (Australia)



Application: Bore Water Hydrology Allocation



Challenge: Gurobi taking too long to solve



Solution: Expert Insight to restructure
formulation

Overview of customer interaction

Water Corp sent us a model file. By default – 5% gap in 30 minutes.

The customer's goal:

- 1% in 10 minutes would be great
- 1% in 30 minutes would be good; or
- 5% in 5 minutes would help whilst troubleshooting.

After tuning for 35 hours across 12 machines - 2% in 30 minutes.

After Simran provided EXPERT INSIGHTS - 0.03% gap in 60 seconds.

Expert Insight – Reformulation Idea

The model contained a lot of a special type of constraint called *indicator constraints*

$$(x = 1) \Rightarrow \sum c \cdot y \leq b$$

If binary variable x takes the value of 1, then add the constraint $\sum c \cdot y \leq b$

These constraints are nice but sometimes result in slow solves because they are **weak**.

Simran notices:

- Many constraints had the same indicator variable
 - Those indicator variables did not belong in any other constraint just the objective
- Simran found a clever reformulation to exploit this.

Our Experts are here to help!

- Gurobi invests heavily in our awesome support team
- Please use us!



Tool 1: Multi-objective Optimization

Managing conflicting objectives efficiently


Customer: Fonterra (New Zealand)



Application: Organic Dairy Production
Planning



Challenge: Solve speed and numerical
issues



Solution: Use our Multi-objective Tools

| Fonterra have conflicting objectives

Many potentially conflicting objectives during production:

- Maximize Profit
- Minimize Waste
- Satisfy different entity preferences
- More...



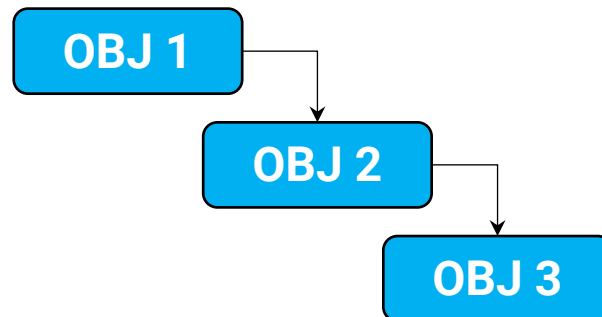
How does Gurobi handle the trade-offs?

- **Weighted:** Optimize a weighted combination of the individual objectives



$$\begin{aligned} \min \quad & w_1 f_1(x) + w_2 f_2(x) + w_3 f_3(x) \\ \text{s.t.} \quad & x \in \mathcal{C} \end{aligned}$$

- **Hierarchical (Lexicographical):** Optimize each objective in a priority order given while limiting the degradation of the higher-priority objectives



$$\begin{aligned} \min \quad & f_1(x) \\ \text{s.t.} \quad & x \in \mathcal{C} \end{aligned}$$

$$\begin{aligned} \min \quad & f_2(x) \\ \text{s.t.} \quad & x \in \mathcal{C} \\ & f_1(x) \leq \epsilon_1 \end{aligned}$$

$$\begin{aligned} \min \quad & f_3(x) \\ \text{s.t.} \quad & x \in \mathcal{C} \\ & f_1(x) \leq \epsilon_1 \\ & f_2(x) \leq \epsilon_2 \end{aligned}$$

- **Weighted + Hierarchical**

How to use multi-objectives

Two options to use multi-objectives:

1. Implement this yourself
2. Use our API

```
Model.setObjectiveN(LinExpr, index, priority=0, weight=1,  
                    abstol=1e-6, reltol=0, name="" )
```

Benefits of using our API:

- Advanced Techniques: Reduced Cost Fixings
- More efficient presolve abilities
- Better numerical behaviour (more consistent behaviour)

Fonterra Case-Study



Fonterra is a global dairy co-operative owned by around 9,000 farmers who produce roughly 30% of the world's dairy exports. The company is committed to sustainable practices.

Industry: Agriculture

Location: Global

Use Cases: Inventory Optimization, Operations, Production, Resource Optimization

Website: www.fonterra.com

Results

- A unified view of key planning problems across the organization that can be addressed with the same solver
- Solve times reduced from 20-40 minutes to roughly one minute

Creating Optimal Product Mixes for Each Day's Unique Dairy Supply

With help from Gurobi, Fonterra uses their new planning model to sustainably produce nearly a third of the world's dairy exports.

As a dairy co-operative owned by roughly 9,000 farmers, Fonterra is committed to sustainable farming practices. But being responsible for 30% of the world's dairy exports while also trying to do right by the environment comes with its own set of challenges, particularly when it comes to planning.

"The dairy industry is almost like the petroleum industry in that you have a 'natural' product with a composition you can't control, and you have to make choices about how to make best use of the components," explained Geoff Leyland, Principal Data Scientist and Head of the Advanced Analytics team at Fonterra.

"Milk composition changes every day, and if one day the milk has more fat than you forecast, you might end up making more butter than you originally planned," he explained.

Because milk is highly perishable, it must be processed within twenty-four hours. And Fonterra's hardest constraint is that

they must process all of the milk in this timeframe.

"That's one of the big challenges of working with a natural product," said Leyland. "We forecast as best we can, but on the day, we don't know exactly how much milk of exactly what composition we will collect, and so we need robust and flexible plans."

Building a Unified System for Optimal Planning

In an effort to address some of those planning problems, Leyland says many teams were using spreadsheets. Also, while they have also used a large-scale product mix model for the last ten years, it no longer suits many of their needs.

"It has become clear that we need to go back to the drawing board and fix things at the foundational level," Leyland noted. "When we were asked to start working on some of the thornier planning problems, that's where Gurobi really managed to do some magic for us."

"Gurobi also made suggestions to fine-tune a production planning LP model, which brought the solve times down from 20-40mins to roughly one minute"

"The support from Gurobi has exceeded our expectations. When we talk to Gurobi, we get smarter."

"We also tried Xpress and CPLEX, and we found that not only is Gurobi faster, but it always finds a solution, which we couldn't get the other solvers to do reliably."

Geoff Leyland, Principal Data Scientist and Head of the Advanced Analytics Team, Fonterra

“ We also tried Xpress

Tool 2: No Relaxation Heuristic

Solving problems previously considered too challenging

Industry: Oil and Gas



Application: Downstream distribution
planning



Challenge: Solvers not making any progress
despite using very large machines



Solution: Use our No Relaxation Heuristic

Sometimes default MIP solve not perfect

Slow root relaxation

Nodes	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
9442783	9.8014026e+10	8.192461e+03	0.000000e+00	39779s				
9451658	9.8014026e+10	7.339784e+03	0.000000e+00	39786s				
9470156	9.8014026e+10	7.289652e+03	0.000000e+00	39793s				
9476934	9.8014026e+10	6.589656e+03	0.000000e+00	39800s				
9488697	9.8014026e+10	6.595012e+03	0.000000e+00	39819s				
9495174	9.8014027e+10	0.000000e+00	0.000000e+00	39903s				
9495280	9.8014027e+10	1.788495e+01	0.000000e+00	39920s				
9495293	9.8014027e+10	0.000000e+00	0.000000e+00	39936s				

Root relaxation: objective 9.801403e+10, 9495293 iterations, 37539.78 seconds (35149.49 work units)

Slow node progress

Nodes	Current Node	Objective Bounds	Work
Expl Unexpl	Obj Depth IntInf	Incumbent BestBd	Gap It/Node Time
0	0	0.00000 0 1762 168.00000 0.00000	100% - 4505s
0	0	-0.00000 0 1623 168.00000 0.00000	100% - 6380s
0	0	-0.00000 0 1776 168.00000 0.00000	100% - 7788s
H 0	0	162.0000000 0.00000	100% - 13016s
0	0	0.00000 0 2751 162.00000 0.00000	100% - 19249s
0	0	0.00000 0 2644 162.00000 0.00000	100% - 23132s
0	0	0.00000 0 2624 162.00000 0.00000	100% - 26440s
0	0	0.00000 0 2480 162.00000 0.00000	100% - 28800s

Slow/no feasible solutions found

Nodes	Current Node	Objective Bounds	Work
Expl Unexpl	Obj Depth IntInf	Incumbent BestBd	Gap It/Node Time
0	0	-5385.7647 0 13339	- - 28s
0	0	-5277.9010 0 15005	- - 81s
...			
0	2	-4610.2942 0 14077	- - 2142s
1	4	-4354.7802 1 14100	- 8993 2149s
3	8	-4116.7369 2 13907	- 5149 2191s
7	12	-4078.5283 3 14300	- 4986 2200s
11	14	-4082.4056 3 14166	- 4514 2215s
15	18	-4064.6967 4 14256	- 3672 2243s
19	22	-3829.7080 4 14160	- 5519 2248s
...			
993	531	-3157.0875 25 13633	- 4185.2719 - 1190 2638s

No improvement in the bound

Nodes	Current Node	Objective Bounds	Work
Expl Unexpl	Obj Depth IntInf	Incumbent BestBd	Gap It/Node Time
0	0	0.00000 0 1762 168.00000 0.00000	100% - 4505s
0	0	-0.00000 0 1623 168.00000 0.00000	100% - 6380s
0	0	-0.00000 0 1776 168.00000 0.00000	100% - 7788s
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0	0	0.00000 0 2644 162.00000 0.00000	100% - 23132s
0	0	0.00000 0 2624 162.00000 0.00000	100% - 26440s
0	0	0.00000 0 2480 162.00000 0.00000	100% - 28800s

Metaheuristics offer a different approach

Examples of metaheuristics

- Ant Colony Optimization
- Genetic Algorithms
- Evolutionary Algorithms
- Particle Swarm Optimization
- Very Large-Neighborhood Search
- Simulated Annealing

Metaheuristics:

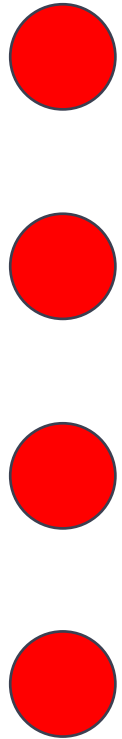
1. Try to find high-quality feasible solutions
2. Can keep running forever

In practice, implementing a metaheuristic yourself can be:

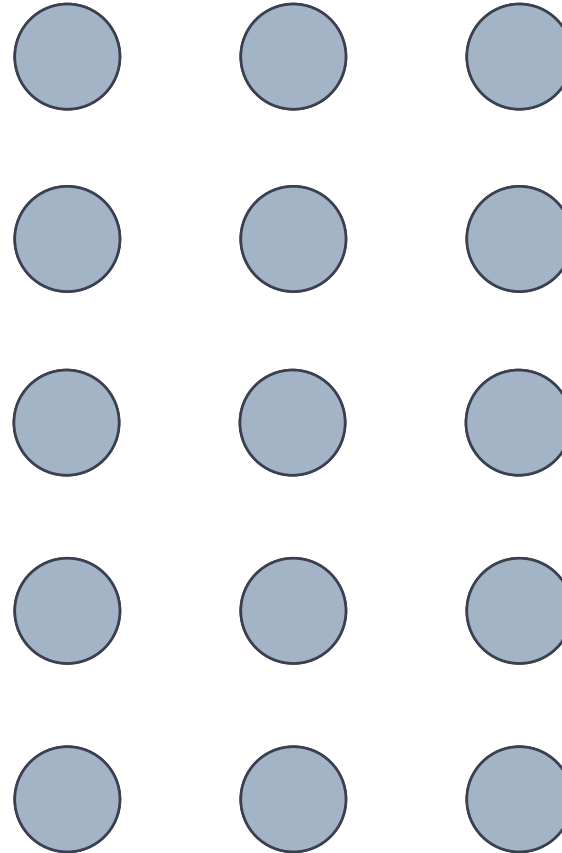
1. Extremely time consuming.
2. Difficult to maintain.
3. Difficult to extend.

We developed our own!

Introducing the No Relaxation Heuristic (NoRel)

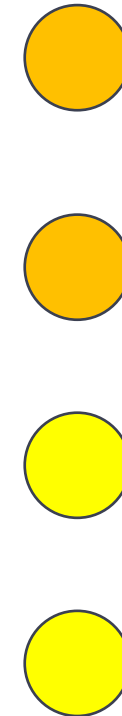


Maintains a pool of
incumbent solutions



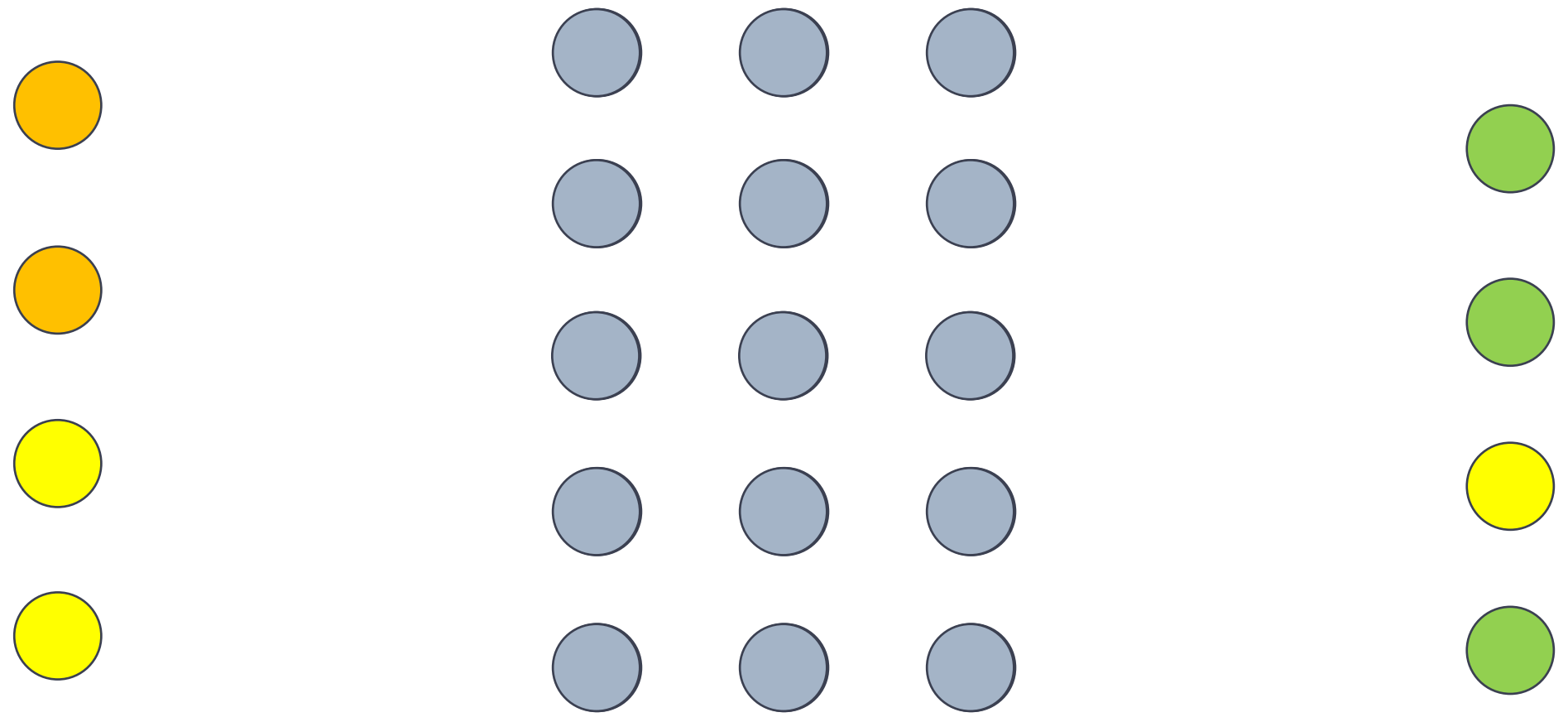
Define neighborhoods
around those solutions

Solve subproblems using
traditional Gurobi solve



Solve proceeds with
new set of incumbents

Introducing the No Relaxation Heuristic (NoRel)



NoRel is extremely powerful in practice

- The approach can be applied to almost all problem types
- To use set the **NoRelHeurTime** parameter
- Parallelizes extremely well for large machines

Tool 3: Bilinear Constraints

Solving the whole problem in the blending industry

Industry: Continuous Manufacturing



Application: Chemical processing

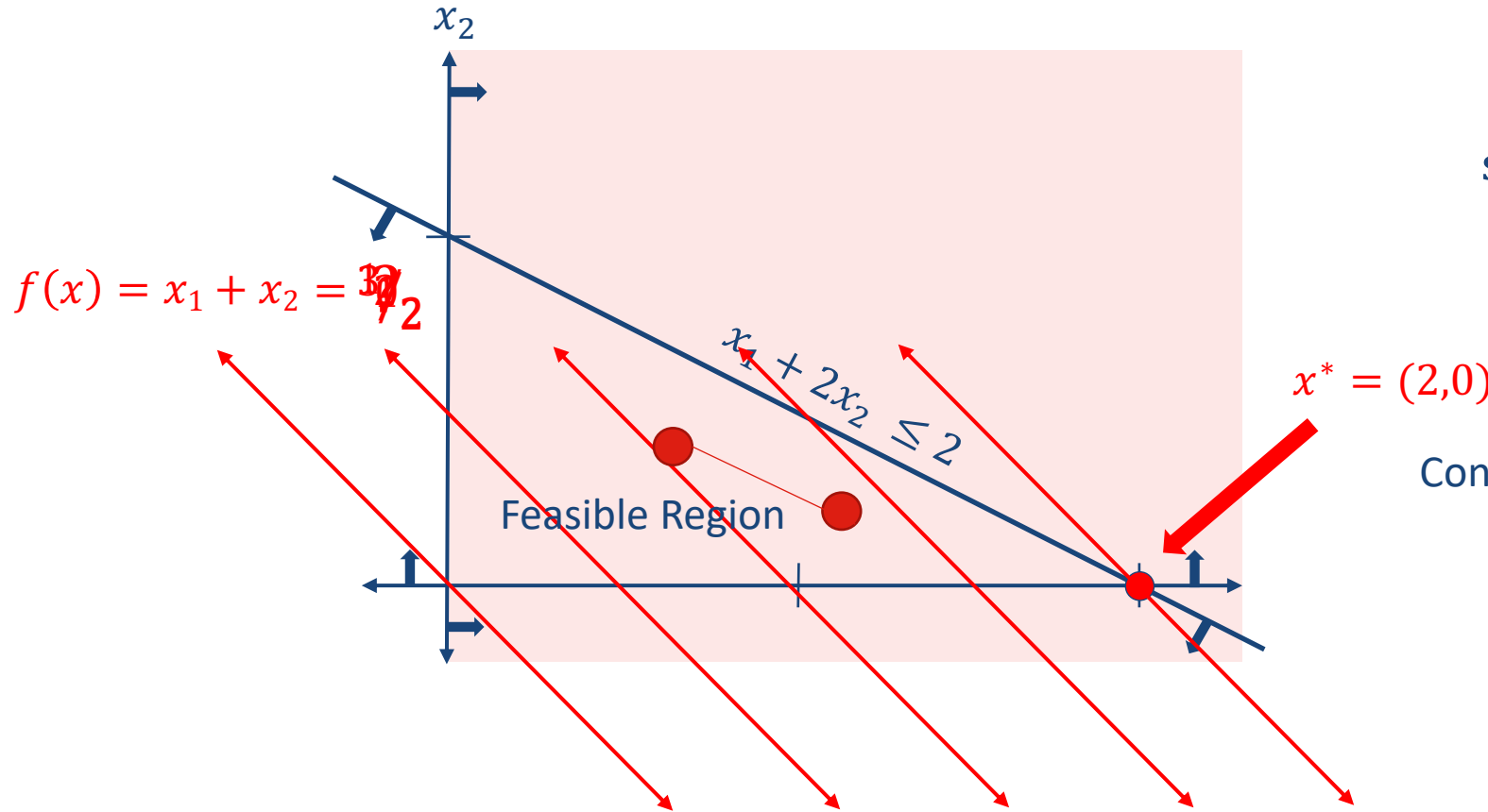


Challenge: Competitor solver unable to model the problem directly.



Solution: Use Gurobi's Bijective Constraints

Most solvers only handle convex constraints



$$\begin{aligned} \max_{x \in \mathbb{R}^2} \quad & x_1 + x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \leq 2 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{aligned}$$

Convexity guarantees solution to be optimal

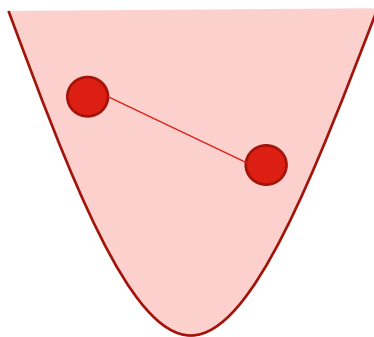
Convexity: A line between any two points in the feasible region, stays within the feasible region

Convex and Non-Convex Quadratics

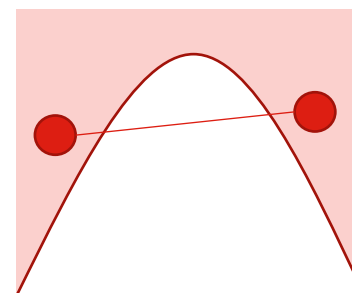
Products of continuous variables

For example, $z = x \cdot y$

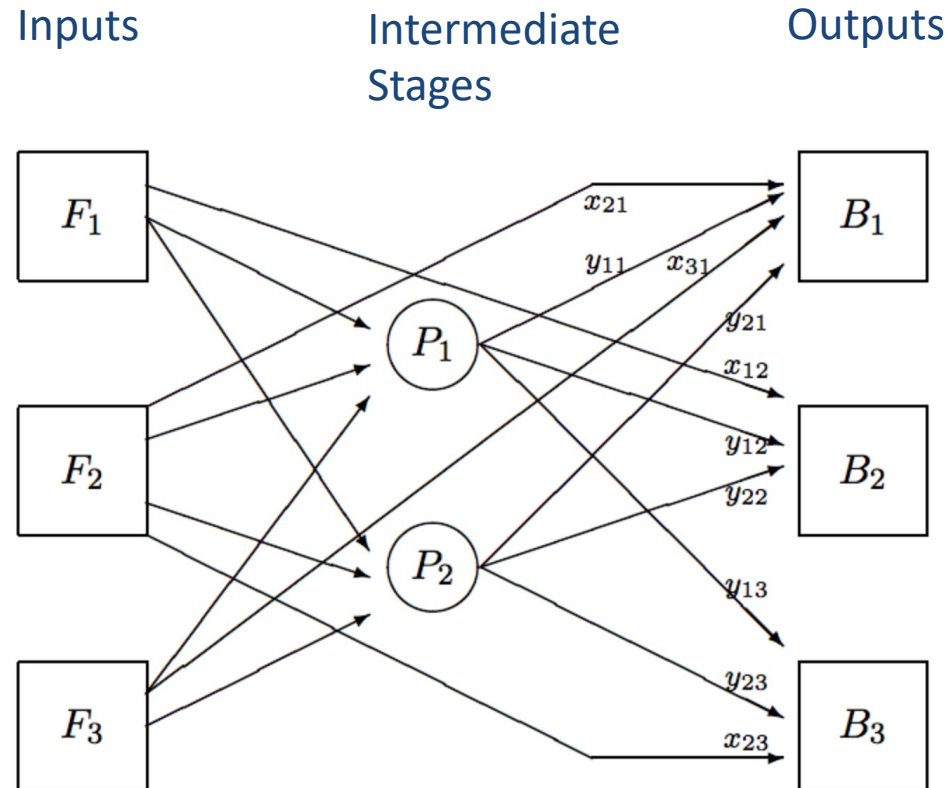
Convex Quadratic



Non-Convex Quadratic



Pooling Problems not possible in other solver



pictures from Costa and Liberti: "Relaxations of multilinear convex envelopes: dual is better than primal"

Goal: Minimize cost of producing target products of a given quality through blending input products

Continuous variables

- Quantity $q_{i,j}$
- Quality p_i

Link quality of tank j with incoming flow:

$$p_j = \frac{\sum_{i \in \delta^-(j)} (p_i q_{i,j})}{\sum_{i \in \delta^-(j)} (q_{i,j})}$$

Summary of interaction

- Customer was using a competitor's solver to solve a pooling problem
- Other solver could not solve non-convex quadratic problems directly.
- Customer had to implement their own heuristic approach as a workaround and obtain sub-optimal solutions
- Even an older version of Gurobi significantly outperformed their approach.

Gurobi Version	9.5.2
MIPGap	68.60%
Objective	\$18,050

- 3-4 times cheaper to make the same set of products
- Less waste! Use inputs of less quality to create the same quality outputs



GUROBI
OPTIMIZATION

Summary

Gurobi has many tools that are extremely useful

- Insights from our Expert Team
- Multi-objective Optimization
- No-Relaxation Heuristic
- Bilinear Constraints
- Global MINLP
- Infeasibility Detection Tools
- Distributed Solving
- Automated Tuning tools
- Piecewise-Linear Approximations
- Machine Learning Tools





Thank You

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